Lecture 4b – Addendum.

Up Gradient Transport → Example

→ Consider coupled heat, particle transport (contrived example)

$$\begin{pmatrix} Q \\ \Gamma \end{pmatrix} = - \begin{bmatrix} \chi, & \alpha \\ \alpha, & D \end{bmatrix} \begin{bmatrix} \nabla T \\ \nabla n \end{bmatrix}$$

$$Q = -\chi \nabla T - \alpha \nabla n$$

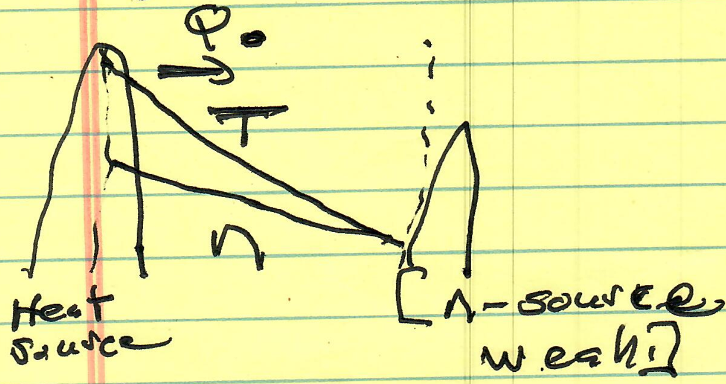
$$\Gamma = -D \nabla n - \alpha \nabla T$$

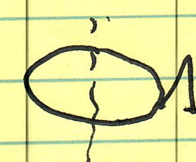
$$\frac{dS}{dt} > 0 \Rightarrow \quad \chi, D > 0$$

$$D\chi - \frac{1}{4}(2\alpha)^2 > 0$$

$$\boxed{D\chi - \alpha^2 > 0}$$

→ Consider





How get peaked density?

$\alpha < 0$! $\Rightarrow$ up-gradient particle flux component.

(but $\alpha^2 < D\chi$)

$\chi D > \alpha^2$ constrains strength up-grad. flux.

steady state, away source:

$$-D\nabla n = \alpha \nabla T$$

$$= -|\alpha| \nabla T$$

$$\boxed{\nabla n = +\frac{|\alpha|}{D} \nabla T}$$

$\nabla T < 0 \Rightarrow \nabla n < 0 \rightarrow$ peaked.

But $D\chi > \alpha^2$ constrains effect of off-diagonal up-gradient flux.

Check: "cost" for temperature?

up-gradient heat flux

$$Q_0 = -\chi \nabla T + |\alpha| \nabla n$$

$$= -\chi \left[1 - \frac{|\alpha|^2}{D\chi} \right] \nabla T$$

origin of constraint!

$$\nabla T = \frac{-Q_0}{\chi \left[1 - \frac{|\alpha|^2}{D\chi} \right]}$$

$$\nabla n = \frac{-|\alpha| Q_0}{D\chi \left[1 - |\alpha|^2 \right]}$$